Fundamental Matrix: Digital camera calibration and Essential Matrix parameters

MARIA LORENA BERGAMINI[1,2], FRANCISCO ANSALDO[1], GLEN BRIGHT[3], JOSÉ FRANCISCO ZELASCO[1]

[1] Facultad de Ingeniería - Universidad de Buenos Aires
Paseo Colón 850 – Ciudad de Buenos Aires
ARGENTINA

[2] CAETI - Facultad de Tecnología Informática – Universidad Abierta Interamericana
Montes de Oca 745 – Ciudad de Buenos Aires
ARGENTINA

[3] University of KwaZulu-Natal
Mazisi Kunene Road - Glenwood - Durban, 4041
SOUTH AFRICA

maria.bergamini@uai.edu.ar
franciscoansaldofiuba@gmail.com
brightg@ukzn.ac.za
jfzelasco@fi.uba.ar

Abstract: The Fundamental Matrix, based on the coplanarity condition, even though it is very interesting for theoretical issues, it does not allow to find the camera calibration parameters, and the base and rotation parameters altogether. In this work there is presented an easy calibration method for calculating the internal parameters: pixel dimensions and image center pixel coordinates. We show that the method is slightly easier if the camera rotation, in relation with the general referential system, is small. We evaluate the accuracy of the four calibration parameters by simulations. When the calibration parameters are known, the Fundamental Matrix can be reduced to the Essential Matrix. In order to find the relative orientation parameters in stereo vision, there is also presented a new method to extract the base and the second (right) camera rotation by means of the Essential Matrix. The proposed method is simple to implement.

Keywords: Fundamental matrix, Essential matrix, camera calibration

1. Introduction

If a point in a 3D referential system is imaged as $\boldsymbol{P_L}$ in the left view (camera 1), and as $\boldsymbol{P_R}$ in the right view (camera 2), then the image point direction vectors in the same 3D referential system satisfy with the base $\boldsymbol{b}$ –vector joining the two camera points of view– the relation $\boldsymbol{P_L} \cdot \boldsymbol{b} \times \boldsymbol{P_R} = 0$. That is, the triple product is zero, because these 3 vectors are in the same plane (epipolar geometry). The skew-symmetric matrix B is obtained from the vector $\boldsymbol{b}$, so that the triple product condition can be stated as $\boldsymbol{P_L} B \boldsymbol{P_R} = 0$.

The pixel coordinates $\boldsymbol{W_L}$ and $\boldsymbol{W_R}$ of both points in digital images, are related to points in length units as the ones used in the 3D referential system, by the calibration matrix. If the left camera determines the 3D referential system, the coordinates in the right camera have to be multiplied by the rotation matrix that accounts for the right camera orientation. Then, $\boldsymbol{W_L} {C^{-1}}^T BRC^{-1} \boldsymbol{W_R} = 0$. The Fundamental Matrix [1,2] is, then, $F = {C^{-1}}^T BRC^{-1}$. It is a 3 × 3 matrix of rank 2, and, even if it is very interesting for theoretical issues related to the epipolar geometry, it is not possible to obtain the 9 parameters involved: 4 parameters for the camera calibration, 3 for the rotation and 2 for the base (known up to scale).

When the calibration is known, only the parameters of B and R are unknown. Then, the Essential Matrix is $E = BR$ [3]. There are several solutions to obtain the 5 involved parameters [4,5,6,7]. The usual method involves the singular value decomposition of the essential matrix [8].

There are some methods to obtain the calibration's of the calibration. What we propose here is a very simple calibration scheme, which is also easy to implement. We assume that it is quite easy to measure the X and Y coordinates of the camera point of view in relation to a 3D referential system when the camera optical axis has a reasonable angle with the Z axis. We then accept that the error in the Z coordinate of the point of view is less than +/- 1 cm. Under these assumptions we evaluate the error in the calibration parameters obtained. Nevertheless, in a later step, assuming that two cameras have a small angle in relation to the Z axis, the error in the calculation of 3D coordinates due to the calibration parameter errors, only involves the Z coordinate. Then, the calibration parameters error can be evaluated and corrected, knowing the difference in the Z coordinate between two benchmarks in relation to the one calculated by stereoscopy.

Regarding the Essential Matrix [9], we propose an original solution focused on extracting the base B and the rotation R, involving the solution of linear systems.

The rest of the paper is organized as follows. In section 2 the method to calculate the camera calibration parameters is exposed. Section 3 describes the calculation of position parameters (base and rotation) from the Essential Matrix. Then, in section 4, the numerical simulations to evaluate accuracy of the calibration parameters are shown. Section 5 presents the conclusions and future works.

2. Camera Calibration.

Let $\boldsymbol{W} = (u, v, 1)^T$ be the pixel coordinates of a point in a digital image, and $\boldsymbol{V} = (x, y, 1)^T$ the image coordinates in length units. They are related by

$$\begin{aligned} u &= \alpha x + u_0 \\ v &= \beta y + v_0 \end{aligned} \tag{1}$$

where (u_0,v_0) is the image principal point, intersection of optical axis to the image, and $\alpha = f.d_x$ and $\beta = f.d_y$ are focal lengths in pixels, product of focal distance by scale factors in horizontal and vertical directions.

This relationship can be stated as

$$\boldsymbol{W} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{bmatrix} \alpha & 0 & uo \\ 0 & \beta & vo \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = C.\boldsymbol{V} \tag{2}$$

The matrix

$$C = \begin{bmatrix} \alpha & 0 & uo \\ 0 & \beta & vo \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

is the intrinsic parameter matrix.

Then,

$$\boldsymbol{V} = C^{-1}.\boldsymbol{W} = \begin{bmatrix} 1/\alpha & 0 & -uo/\alpha \\ 0 & 1/\beta & -vo/\beta \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \tag{4}$$

Let $\boldsymbol{P} = (X, Y, Z)^T$ be the coordinates of a point in the space, with respect to a fixed 3D referential system. Let us assume a camera in the point of coordinates $\boldsymbol{b} = (X_c, Y_c, Z_c)^T$, and rotated respect to the reference; and this rotation is given by a rotation matrix R. Thus,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \lambda R C^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \tag{5}$$

Without loss of generality, let us assume $\boldsymbol{b} = (0,0,0)^T$.

Now, let us name $RC^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Then,

$$\begin{aligned} X &= (a_{11}u + a_{12}v + a_{13})\lambda \\ Y &= (a_{21}u + a_{22}v + a_{23})\lambda \\ Z &= (a_{31}u + a_{32}v + a_{33})\lambda \end{aligned} \quad (6)$$

From here we take,

$$\begin{aligned} \frac{X}{Y} &= \frac{a_{11}u + a_{12}v + a_{13}}{a_{21}u + a_{22}v + a_{23}} \\ \frac{X}{Z} &= \frac{a_{11}u + a_{12}v + a_{13}}{a_{31}u + a_{32}v + a_{33}} \\ \frac{Y}{Z} &= \frac{a_{21}u + a_{22}v + a_{23}}{a_{31}u + a_{32}v + a_{33}} \end{aligned} \quad (7)$$

Reordering,

$$\begin{aligned} & a_{21}uX + a_{22}vX + a_{23}X - a_{11}uY - a_{12}vY - a_{13}Y = 0 \\ & a_{31}uX + a_{32}vX + a_{33}X - a_{11}uZ - a_{12}vZ - a_{13}Z = 0 \\ & a_{21}uZ + a_{22}vZ + a_{23}Z - a_{31}uY - a_{32}vY - a_{33}Y = 0 \end{aligned} \quad (8)$$

Having a set of scene points whose coordinates are known, $\boldsymbol{P_k} = (X_k, Y_k, Z_k)$ projected to $(u_k, v_k, 1)$ in the image, the equations (8) constitute a linear system of equations for the unknowns a_{ij}.

The system is homogeneous and the matrix has one-dimension kernel. The solution can be calculated up to scale factor:

$$\begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{31} \\ a_{32} \\ a_{33} \end{pmatrix} = \rho \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \end{pmatrix} \quad (9)$$

where the s_i are obtained by solving the system, and ρ is a scale factor. Therefore,

$$RC^{-1} = \begin{bmatrix} r_{11}/\alpha & r_{12}/\beta & r_{13} - \frac{r_{11}u_o}{\alpha} - \frac{r_{12}v_0}{\beta} \\ r_{21}/\alpha & r_{22}/\beta & r_{23} - \frac{r_{21}u_0}{\alpha} - \frac{r_{22}v_0}{\beta} \\ r_{31}/\alpha & r_{32}/\beta & r_{33} - \frac{r_{31}u_0}{\alpha} - \frac{r_{32}v_0}{\beta} \end{bmatrix} = \rho \begin{bmatrix} s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 \\ s_7 & s_8 & s_9 \end{bmatrix} \quad (10)$$

Now, it is evident that

$$\begin{aligned} r_{21} &= r_{11}\frac{s_4}{s_1} \quad r_{31} = r_{11}\frac{s_7}{s_1} \\ r_{22} &= r_{12}\frac{s_5}{s_2} \quad r_{32} = r_{12}\frac{s_8}{s_2} \end{aligned} \quad (11)$$

Since R is an orthogonal matrix, its columns are orthogonal vectors. Then, we can assert

$$r_{11}^2 + r_{21}^2 + r_{31}^2 = r_{11}^2\left(1 + \left(\frac{s_4}{s_1}\right)^2 + \left(\frac{s_7}{s_1}\right)^2\right) = 1$$

$$r_{11}^2 = \frac{s_1^2}{s_1^2 + s_4^2 + s_7^2} \quad (12)$$

Similarly for r_{12}

$$r_{12}^2 = \frac{s_2^2}{s_2^2 + s_5^2 + s_8^2} \quad (13)$$

At this point, two columns of R, R_{*1} and R_{*2} are known. The third column can be calculated since it must be orthogonal to R_{*1} and R_{*2}. Then, $R_{*3} = R_{*1} \times R_{*2}$.

Knowing R, the calibration matrix arises as

$$C^{-1} = \begin{bmatrix} \frac{1}{\alpha} & 0 & -\frac{u_o}{\alpha} \\ 0 & \frac{1}{\beta} & -\frac{v_0}{\beta} \\ 0 & 0 & 1 \end{bmatrix} = \rho R^{-1} \begin{bmatrix} s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 \\ s_7 & s_8 & s_9 \end{bmatrix} \quad (14)$$

Clearly, the value of ρ is the one that makes the (3,3) element of C^{-1} equal to 1.

Admitting that point measurement has always errors, it is convenient to use more points to over determining system (8), and solve it by means of least square minimization.

2.1. Approximation for small rotation

Consider the rotation angle small. In this case, the rotation matrix can be approximated with

$$R \approx \begin{bmatrix} 1 & -w_z & w_y \\ w_z & 1 & -w_x \\ -w_y & w_x & 1 \end{bmatrix} \tag{15}$$

Thus,

$$RC^{-1} \cong \begin{bmatrix} \dfrac{1}{\alpha} & -\dfrac{w_z}{\beta} & -\dfrac{u_0}{\alpha}+\dfrac{w_z v_0}{\beta}+w_y \\ \dfrac{w_z}{\alpha} & \dfrac{1}{\beta} & \dfrac{w_z u_0}{\alpha}-\dfrac{v_0}{\beta}-w_x \\ -\dfrac{w_y}{\alpha} & \dfrac{w_x}{\beta} & \dfrac{w_y u_0}{\alpha}-\dfrac{w_x v_0}{\beta}+1 \end{bmatrix} \tag{16}$$

From where,

$$w_z = \frac{s_4}{s_1}, w_y = -\frac{s_7}{s_1}, w_x = \frac{s_8}{s_5} \tag{17}$$

And consequently R is known. Then, the calibration coefficients came from (14).

2.2. Calibration Platform

The 3D general reference system consists of a vertical grid whose axis X is horizontal. The optical axis of the camera is placed pointing to the grid. The distance between the camera and the grid may be around 1m or 1.5 m.

With a length greater than the distance from the camera to the grid, we join the center of the camera with the grid in two symmetrical points, in vertical way and again in horizontal way. The midpoint of these four points determines the X and Y coordinates of the optical center of the camera.

Since we do not know with certainty the position of the optical center within the camera, we know *approximately* the Z coordinate of the optical point of view. This incertitude will cause an error in the calibration parameters.

At section 4 we have estimated by simulation these errors in the calibration parameters.

In a later work, we will present how to correct these calibration parameters. We will compare the distance in depth between two benchmarks against the one obtained by stereoscopy using the calibration values.

3. Essential Matrix parameters.

The essential matrix allows to express the relation between corresponding points in a pair of stereo images from calibrated cameras.

Fixing a 3D referential system in left camera, with Z-axis parallel to its optical axis, let us denote $\boldsymbol{b} = (X_c, Y_c, Z_c)^T$ the coordinates of the right camera with respect to this reference, and R the rotation matrix that gives the orientation of right camera.

As it was stated before, $\boldsymbol{V_L}$, $R\boldsymbol{V_R}$ and $\boldsymbol{b}$ are coplanar, where $\boldsymbol{V_L}$ and $\boldsymbol{V_R}$ are the coordinates of corresponding image points. Therefore,

$$\boldsymbol{V_L} \cdot (\boldsymbol{b} \times R\boldsymbol{V_R}) = \boldsymbol{V_L^T} BR\boldsymbol{V_R} = 0 \tag{18}$$

Here, B is the skew-symmetric matrix corresponding to $\boldsymbol{b}$, that is,

$$B = \begin{pmatrix} 0 & -Z_c & Y_c \\ Z_c & 0 & -X_c \\ -Y_c & X_c & 0 \end{pmatrix} \tag{19}$$

$E = BR$ is the essential matrix. E can be determined up to scale, knowing 8 pairs of homologous points.

Thus, it can be calculated $\tilde{E} = \lambda E = \lambda BR$, where λ is an unknown scale factor. The columns of E are orthogonal to $\boldsymbol{b}$. Then, the cross product of two columns of E is a vector parallel to $\boldsymbol{b}$. Let $\boldsymbol{b}'$ be a unit vector parallel to $\boldsymbol{b}$;

$$\frac{\tilde{E}_{*1} \times \tilde{E}_{*2}}{\left|\tilde{E}_{*1} \times \tilde{E}_{*2}\right|} = \boldsymbol{b}' = \gamma \boldsymbol{b} \tag{20}$$

then $\gamma = \frac{1}{|\boldsymbol{b}|}$. Let denote B' the skew-symmetric matrix related to $\boldsymbol{b}'$.

The matrix

$$\tilde{E}^T\tilde{E} = \lambda^2 R^T B^T BR = R^T(\lambda^2 B^T B)R \tag{21}$$

is similar to $\lambda^2 B^T B$, and therefore they have the same trace,

$$\mathrm{tr}\left(\tilde{E}^T\tilde{E}\right) = \lambda^2 \mathrm{tr}\left(B^T B\right) = \lambda^2 2|\mathbf{b}|^2 \tag{22}$$

From this,

$$\lambda = \pm \frac{\sqrt{\mathrm{tr}(\widetilde{\mathrm{E}}^{\mathrm{T}}\widetilde{\mathrm{E}})}}{\sqrt{2}|\boldsymbol{b}|} \tag{23}$$

Then,

$$\widetilde{\mathrm{E}} = \lambda \frac{\mathrm{B}'}{\gamma}\mathrm{R} = \pm \frac{\sqrt{\mathrm{tr}(\widetilde{\mathrm{E}}^{\mathrm{T}}\widetilde{\mathrm{E}})}}{\sqrt{2}}\mathrm{B}'\,\mathrm{R} \tag{24}$$

Thereafter,

$$\pm\widetilde{\mathrm{E}}\frac{\sqrt{2}}{\sqrt{\mathrm{tr}(\widetilde{\mathrm{E}}^{\mathrm{T}}\widetilde{\mathrm{E}})}} = \mathrm{E}' = \mathrm{B}'\,\mathrm{R} \tag{25}$$

Now, E' and B' are known. Since B' is singular, it cannot be inverted to have $= (B')^{-1}E'$. However, being R an orthogonal matrix, it can be calculated as follows, according to literature.

E admits a singular value decomposition $E = USV^T$, where U and V are orthogonal matrices and S is a diagonal matrix with the singular values of E in its diagonal. Then, B and R are obtained as,

$$B = UASU^T \tag{26}$$

$$R = UAV^T \tag{27}$$

with $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
There is also another solution $R = UA^TV^T$.

Since there is a sign ambiguity in E from (25), there are other two possible solutions for the factorization B-R, using the opposite sign.

Another original way of extracting B and R from E, without singular value decomposition, is as follows.

Each column of R satisfies the equation $E'_{*i} = B'R_{*i}$. Since B' has rank 2, the solution space is one-dimensional. Solving it, $R_{*i} = R^0_{*i} + \gamma_i N$, where N is a unit vector in the null space of B' (that is, a vector such that $B'N = 0$) and R^0_{*i} is a particular solution of the system.

Then,

$$R = [R_{*1} \quad R_{*2} \quad R_{*3}] = [R^0_{*1} \quad R^0_{*2} \quad R^0_{*3}] + N[\gamma_1 \quad \gamma_2 \quad \gamma_3] \tag{28}$$

for some scalars γ_i. The objective now is to calculate these scalars.

The orthogonality of R implies $R^TR = I$, that is $R^T_{*i}\, R_{*j} = 0$ if $i \neq j$; and $R^T_{*i}\, R_{*i} = 1$.

From this point,

$$1 = R^T_{*i}R_{*i} = {R^0_{*i}}^T R^0_{*i} + 2\gamma_i {R^0_{*i}}^T N + \gamma_i^2 N^T N = |R^0_{*i}|^2 + 2\gamma_i {R^0_{*i}}^T N + \gamma_i^2 |N|^2 \tag{29}$$

Using that N is a unit vector, the possible values of γ_i are

$$\gamma_i = -{R^0_{*i}}^T N + \sqrt{({R^0_{*i}}^T N)^2 + 1 - |R^0_{*i}|^2} \tag{30}$$

$$\gamma_i = -{R^0_{*i}}^T N - \sqrt{({R^0_{*i}}^T N)^2 + 1 - |R^0_{*i}|^2} \tag{31}$$

Orthogonality of R also implies

$$0 = R^T_{*i}R_{*j} = {R^0_{*i}}^T R^0_{*j} + \gamma_i N^T R^0_{*j} + \gamma_j {R^0_{*i}}^T N + \gamma_j \gamma_i \tag{32}$$

for i,j=1,2,3, $i > j$.

With the values of γ_i found previously in (30) and (31), equation (32) takes the form:

$$\pm\sqrt{({R^0_{*i}}^T N)^2 + 1 - |R^0_{*i}|^2}\sqrt{({R^0_{*j}}^T N)^2 + 1 - |R^0_{*j}|^2} = ({R^0_{*i}}^T N)({R^0_{*j}}^T N) - {R^0_{*i}}^T R^0_{*j} \tag{33}$$

Here, the + sign corresponds to take (30) for both i and j, or (31) for both i and j; and the – sign corresponds to take (30) for i and (31) for j, or vice versa.

The strategy is then, to calculate $({R^0_{*i}}^T N)({R^0_{*j}}^T N) - {R^0_{*i}}^T R^0_{*j}$. If it is positive, γ_i and γ_j are calculated both with (30) or both with (31). And if it is negative, γ_i is calculated with (30) and γ_j is calculated with (31), or vice versa.

Thus, two possible sets of values $(\gamma_1, \gamma_2, \gamma_3)$, are obtained, for each choice of sign in (25). Two orthogonal matrices R are obtained; one of them is a direct orthogonal matrix R_1 (it has determinant 1; pure rotation) and the other is an inverse orthogonal matrix R_2 (it has determinant -1; rotation and symmetry). Then, $-R_2$ is a direct orthogonal matrix; and it is one of the matrices that would be obtained assuming opposite sign in (25).

Summarizing, four orthogonal matrices, R_1, $-R_1$, R_2 and $-R_2$, satisfy $\pm\widetilde{\mathrm{E}}\frac{\sqrt{2}}{\sqrt{\mathrm{tr}\,(\widetilde{\mathrm{E}}^{\mathrm{T}}\widetilde{\mathrm{E}})}} = \mathrm{B}'\mathrm{R}$, but only two of them are rotation matrices. Both are valid solutions of the system. But one of them predicts points behind the camera.

4. Simulation: Evaluating accuracy of the calibration parameters

Numerical simulations were performed, in order to evaluate the accuracy of the calibration method proposed, and the sensitivity to the Z_c coordinate (which in practical situations is hard to determine with precision, as it was explained before).

A simulated camera was defined, using parameter values α=400, β=380, u_0=600 and v_0=500. It was assumed that the camera orientation is rotated an angle of $\pi/50$ about the direction $(1,1,1)^T$. A set of 9 points in a vertical plane were considered for calibration scheme.

Table 1 shows the results of the numerical simulation. Column 2 to 5 show the calculated values of the intrinsic parameters, considering certain error in the value of $Z_c = 100 + \varDelta Z_c$.

Table 1: Numerical results from simulation

Parameter	$\varDelta Z_c$			
	-0.1	0.1	-1	1
α	399.6	400.4	396.01	403.98
β	379.6	380.4	376.2	383.78
u_0	599.97	600.03	599.71	600.28
v_0	500.03	499.97	500.23	499.72

Assuming the simplified rotation matrix, valid for small rotation, the parameter estimated values are very close to the one obtained with full rotation matrix. Table 2 shows the results.

Table 2: Numerical simulation for small rotation

Parameter	$\varDelta Z_c$			
	-0.1	0.1	-1	1
α	399.5	400.4	396.0	403.9
β	379.6	380.4	376.18	383.7
u_0	599.4	599.4	599.14	599.7
v_0	499.5	499.5	499.77	499.2

In practice, points in the space are measured with certain errors. In the next experimental analysis, noise was added to the measured points. Column 2 and column 3 show the parameters obtained when points have errors of 0.1% and 1% respectively.

Table 3: Parameters obtained with noisy data

Parameter	Error 0.1%	Error 1%
α	399.69	397.35
β	379.38	379.76
u_0	600.33	609.3
v_0	500.15	495.3

As it can be seen, quite accurate estimation was obtained, even admitting imprecise data within reasonable limits.

5. Conclusion and perspectives

Both methods, one to obtain the 4 calibration parameters and the other to obtain the 5 Essential Matrix parameters are original and simple. In particular for the Essential Matrix we noted an error that appears in proposed solutions of some manuscripts and courses notes, that is to consider that B can be inverted.

Also we may notice that when the Essential Matrix parameters are obtained, if the cameras have a small rotation in relation with the referential system, the difference between two aligned points in the Z direction is proportional to the calibration error. In a future work we will evaluate this issue to improve the accuracy of the calibration parameters to obtain more precise 3D coordinates of the scene points obtained by stereoscopy.

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